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Conductive HET Flow in the HET Dome

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I. Fourier's Law for Heat Flow

Conductive heat flow is calculated using "Fourier's Law:"

$$\vec{q} \;=\; -k\nabla T \tag{1}$$

where ∇T is the gradient of the temperature in the body, k is a constant called "thermal conductivity," the vector $\vec{q}$ is the energy flow per unit time per unit area, and the negative sign makes heat flow from regions of high temperature to regions of low temperature. For one-dimensional flow in the x direction, equation 1 simplifies to

$$q \;=\; -k\frac{\partial T}{\partial x} \tag{2}$$

where q is the (scalar) heat flow in the x direction. Equations 1 and 2 assume that the temperature distribution is already known. Solving for the temperature distribution from first principles is a much more difficult problem.

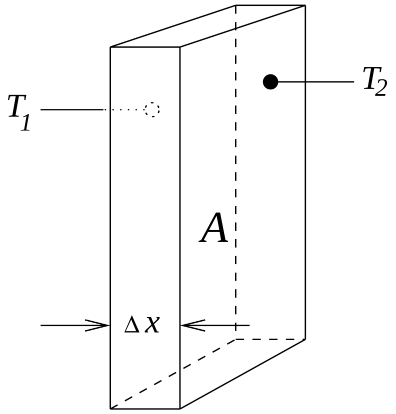

Figure 1: Heat flow through a slab. The slab has thickness Δx, area A, and temperatures T_1 and T_2 on its two faces.

Consider a slab with thickness Δx, area A, and temperatures T_1 and T_2 on its two faces (see Figure 1). The mean total heat flow through the slab is given by the integral of equation 2 over x and A:

$$\dot{Q} \;=\; \frac{1}{\Delta x}\int_x\int_A q\,dxdA \;=\; \frac{1}{\Delta x}\int_x\int_A -k\frac{\partial T}{\partial x}dxdA \tag{3}$$

where $\dot{Q}$ is the energy flowing through the slab per unit time (eg, Watts). If the slab is uniform so that k is not a function of x, equation 3 takes the simple form:

$$\dot{Q} = \frac{kA}{\Delta x}(T_1 - T_2) \tag{4}$$

II. R-values and the Working Equation for Heat Flow

For convenience, one replaces the quantity $k/\Delta x$ in equation 4 by the R-value of the slab, where the R-value is defined to be

$$R = \frac{\Delta x}{k} \tag{5}$$

so that

$$\dot{Q} = \frac{A}{\mathrm{R}}(T_1 - T_2) \tag{6}$$

Equation 6 is the basic working equation for many simple problems in heat flow.

R-values are typically given in English units (yuck); the conversion factor from English units to metric units is 0.176228. Some useful R-values are given in Table 1 along with their metric equivalents. R-values add: A slab with two layers whose R-values are R_1 and R_2 has a total R-value $\mathrm{R} = \mathrm{R}_1 + \mathrm{R}_2$.

Table 1: Some Useful R-values

Material	R (English)	R (Metric)*
8” Mineral wool blanket and batt	30	5.3
Expanded polyurethane	6.25/inch	1.10/inch
1” plywood	1.24	0.219
1/4” hardboard (Masonite)	0.31	0.055
Tile	0.05	0.001
Poured Concrete	0.08/inch	0.014/inch
1 1/2” Soft Wood	1.25	0.220
Air films		
Inside (ceiling or wall)	0.61	0.107
Outside (7.5 mph wind)	0.25	0.044
Outside (15 mph wind)	0.17	0.030
Air Space (1/2” to 4” trapped air)	1.00	0.176228

*Metric units: Kelvin Meter^2 per Watt

The physics of heat flow from a wall to air is complicated but can still be calculated approximately using equation 6. As shown in Figure 2, the energy in the wall must pass through a boundary layer before it can arrive in turbulent, moving air and be carried away. The boundary layer is a thin film in which air is stagnate immediately next to the wall and has laminar flow further away from the wall. Heat flows through the boundary layer only by conduction, so the layer acts as an insulator.

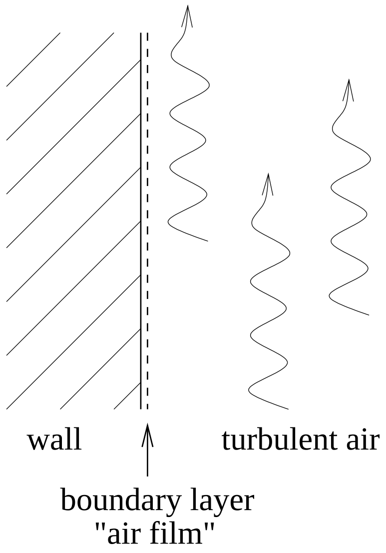

Figure 2: Heat flow from a wall to air. The wall and the turbulent air are sources and sinks of heat. There is a thin boundary layer (an air film) next to the wall where the air is either stagnate of has laminar flow. Heat flows through the boundary layer only by conduction.

The properties of the boundary layer and, therefore, its R-value depend on local conditions. Inside buildings where air flow is slow, the boundary layer is thicker ($\sim$ 1/4") and has a fairly high R-value. The boundary layer is thinner on outside walls exposed to winds ($\sim$ 1/10") and has a smaller R-value.

III. Applications

The Dome: We assume that the interior surface of the dome is spherical and smooth. The surface area of the dome, A_{dome}, is

$$A_{dome} = 2\pi R_{dome}^2(1 - \cos\theta) - \pi R_{aper}^2 \tag{7}$$

where R_{dome} is the radius of the dome, θ is the angle of the lower edge of the dome as seen from the dome's center, and R_{aper} is the area of the aperture in the dome. The approximate values of the relevant quantities are:

$$\begin{aligned} R_{dome} &\approx 12.2 \text{ m} \\ \theta &\approx 112^\circ \\ R_{aper} &\approx 6.0 \text{ m} \end{aligned} \tag{8}$$

and so the area of the dome is

$$A_{dome} \approx 1286 - 137 = 1149 \text{ m}^2 \tag{9}$$

The interior surface of the dome is well ventilated and acts as if it is an outside wall swept by a low-velocity wind. From Table 1 the correct R-value for these conditions is R = 0.044. The rate of heat flow from the inner surface of the dome is, then,

$$\begin{aligned} \dot{Q}_{dome} &= \frac{A}{\text{R}}(T_1 - T_2) = \frac{1149}{0.044}(T_1 - T_2) \\ &= 26,000\ (T_1 - T_2) \text{ Watts} \end{aligned} \tag{10}$$

The dome radiates into the night sky, typically cooling several degress below the ambient air temperature. Since the dome is made of uninsulated aluminum, the interior surface of the dome also cools below the outside air temperature. From thermal images obtained by E. Barker, much of the dome is a few degrees Farenheit colder that the outside air, so we take $T_1 - T_2 \approx -1°C$. The heat flow from the inner surface of the dome is

$$\dot{Q}_{dome} = -26,000 \text{ Watts} \tag{11}$$

Note that the dome is *cooling* the air inside the dome and, furthermore, this cool air is being produced above the telescope. Some of the cold air probably falls directly into the telescope beam, but some probably flows down along the inner wall of the dome until it comes to the open louvers in the dome wall, where it is picked up by wind flowing through the louvers and carried across the primary mirror.

The approximation used in this calculation did not include the effects of the dome's support ribs. The ribs have significant surface area and act as cooling vanes, increasing the cooling by the interior surface of the dome. The extra cooling is difficult to calculate but could easily add another 10% – 25% to the cooling.

M3 Engineering and Technology corporation calculated that the dome *adds* 5,520 Watts of heat to the dome interior, a gross error.

Concrete in the Dome – A Bogus Calculation: We first calculate the heat flow assuming that all the concrete is uninsulated, including the dome floor. The surface area of the concrete in the dome is about 525 m^2 (see Table 2).

Table 2: Surface Areas of Concrete in the Dome

Location	Surface Area
Floor outside the pier	282 m^2
Floor inside the pier	133 m^2
Telescope Pier	110 m^2

With the dome louvers open, the air flow through the dome is similar to the air flow for exterior walls, so we take R = 0.044. The heat flow from the concrete is, then,

$$\begin{aligned}\dot{Q}_{BareConcrete} &= \frac{A}{\mathrm{R}}(T_1 - T_2) \;=\; \frac{525}{0.044}(T_1 - T_2) \\ &= 12,000\ (T_1 - T_2)\ \text{Watts} \qquad (12)\end{aligned}$$

If the concrete lags the ambient air temperature by $1°C$, the heat flow into the dome is

$$\dot{Q}_{BareConcrete} \;=\; 12,000\ \text{Watts} \qquad (13)$$

of which 2500 Watts come from the telescope pier. This calculation is bogus because (1) much of the concrete is insulated and (2) the assumption of a constant 1° C temperature difference is unjustified.

Conduction of Heat into the Dome from the Ground and the Basement: Heat can be conducted into the dome through the floor and the telescope pier from the ground and the basement rooms.

The dome floor and the telescope pier are made of poured concrete. The dome floor inside the telescope pier lies above the basement spectrograph room. It honeycombed and most of it is only 3 1/2” thick. The dome floor outside the telescope pier lies above either basement rooms (1/4 of the area) or dirt fill (3/4 of the area). This part of the dome floor is 8” thick. The surface areas of the dome floor above the basement rooms and above dirt fill are given in Table 3.

Table 3: Surface Areas, R-Values, and A/R-Values of the Floor and Pier

Location	Surface Area	R-value	A/R
Floor above dirt fill	212 m^2	3.1	68
Floor above basement rooms	203 m^2	2.1	97
Telescope Pier	110 m^2	1.3	85

The dome floor is covered by multi-layer insulation (see Figure 3); the R-values for the individual layers are given in Table 4. The parts of the dome floor over basement rooms have additional insulation from 3 1/2” of concrete and an interior air film at the ceiling of the basement rooms. The parts of the dome floor over dirt fill have additional insulation from 8” of concrete. The dirt fill was originally uncompacted and has almost certainly settled since the dome was built, leaving a stagnate air gap just below the concrete, which acts as a good insulator. For parts of the dome floor above basement rooms the total R-value is 2.1 and for parts of the floor above dirt fill the R-value is 3.1.

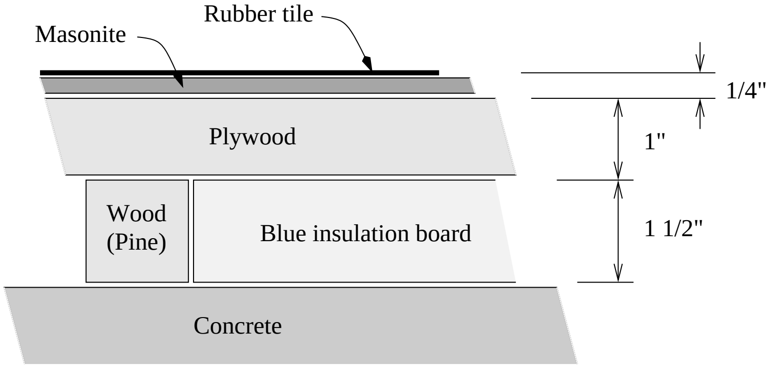

Figure 3: The insulation on the dome floor.

Table 4: R-values for the Insulated Dome Floor

Layer	R-value (above basement)	R-value (above dirt fill)
Outside air film (7.5 mph)	0.044	0.044
Rubber tile	0.001	0.001
1/4” Masonite	0.055	0.055
1” Plywood	0.219	0.219
1 1/2” Insulating board	1.650	1.650
3 1/2” Poured concrete	0.049	–
8” Poured Concrete	–	0.112
Inside air film	0.107	–
Air space (over settled fill)	–	1.000
Total:	2.125	3.081

The R-value of the telescope pier is difficult to estimate because the geometry is complicated and heat can flow into the pier from both the basement rooms and the dirt fill. Assume that heat from the ground and the basement rooms must flow through about about 2 meters (80 inches) of concrete to reach air in the dome. Two meters of concrete plus one external air film and one internal air film yield an R-value of about 1.3. The R-values and the values of A/R for the three main conducting regions are given in Table 3.

Assume that the temperature in the basement rooms and the temperature in the dirt fill are both near 15° C. The heat flow through the dome floor and telescope pier is, then,

$$\dot{Q} \;=\; 250\,(15 - T)\ \text{Watts} \tag{14}$$

where T is the outside air temperature. In the summer time the temperature of the outside air is typically 15° ± 5° C. In the winter the temperature of the outside air is typically

$5^\circ \pm 5^\circ$ C. The heat flow into the dome is

$$\begin{aligned} \text{Summer:} \quad \dot{Q} &= 0 \pm 1300 \text{ Watts} \qquad (15) \\ \text{Winter:} \quad \dot{Q} &= 2500 \pm 1300 \text{ Watts} \qquad (16) \end{aligned}$$

Two thirds of this heat flow is coming through the telescope pier and the floor above the basement rooms.

Heating by the Thermal Inertia of the Dome Floor and Telescope Pier: From previous sections, the R-value for the entire dome floor is 2.00 and the R-value for the pier = 0.044. The area of the floor is 415 m^2 and the area of the pier is 110 m^2.

It is far from clear what temperature difference to take for calculating the effects of thermal inertial, but let us take a worst case calculation: Assume that the concrete under the insulation on the floor lags the outside air temperature by as much as 10° C. The concrete in the pier lags the outside air temperature by as much as 3° C. These conditions could occur during times of rapid change in the temperature of the ambient air, for example during twilight and evening cool down. See Appendix A for calculations justifying the long time scales for the equilibration of concrete, especially insulated concrete.

The heat injected into the dome is, then

$$\begin{aligned} \dot{Q} &= \frac{415}{2.00} \times 10 + \frac{110}{0.044} \times 3 \\ &= 2100 + 7500 \text{ Watts} \\ &= 9600 \text{ Watts} \qquad (17) \end{aligned}$$

Note that the pier is responsible for most of this heating.

If we drop the temperature difference at the pier to 1° to be in accord with the heat flow calculation by M3, we find $\dot{Q} = 2700$ Watts. M3 gives 18,400 Watts from the concrete; again, grossly wrong.

Suppose the pier is covered 2" of expanded polyurethane to give R $\approx$ 2. Adopt winter extreme conditions, so ΔT is 10° C, we have

$$\begin{aligned} \dot{Q} &= \frac{415}{2.00} \times 10 + \frac{110}{2.00} \times 10 \\ &= 2600 \text{ Watts} \qquad (18) \end{aligned}$$

The dome floor is now the dominant source of heat.

Context – The Heat Carried by Air Flowing through the Dome Louvers: The height of the louvers is 16 ft and their width is 14 ft for an area of 224 ft^2. Assume that 4 panels are effectively admitting air to the dome, so the total area of the openings admitting air is 896 ft^2.

A wind speed of 5 mph is 7.33 ft/sec, so at this wind speed the volume of air passing through the dome is

$$\begin{aligned} \frac{dV}{dt} &= 7.333 \times 896 = 6570 \text{ ft}^3/\text{sec} \\ &= 186 \text{ m}^3/\text{sec} \end{aligned} \tag{19}$$

If the air is heated to a uniform temperature, the amount of heat carried away by the flowing air is

$$\dot{Q} = C_a \Delta T \frac{dV}{dt} \tag{20}$$

where C_a is the heat capacity of air at constant pressure, and ΔT is the change in the temperature of the air. Adopting $C_a = 1.0 \times 10^3$ J/m^3/°C and $\Delta T = 0.4$ °C (the maximum that will not significantly degrade seeing), we have:

$$\dot{Q} = 74,000 \text{ Watts} \tag{21}$$

IV. Summary

- The basic equation for conductive heat flow through a uniform slab is

  $$\dot{Q} = \frac{A}{\mathrm{R}}(T_1 - T_2) \tag{22}$$

  where $\dot{Q}$ is the heat flow (Watts in the mks system), A is the cross-sectional area of the slab, T_1 and T_2 are the temperatures on the two sides of the slab, and R is the "R-value" value of the slab.

- Some useful R-values are listed in Table 1. The R-values for heat flow from a wall through a boundary layer (an air film) into turbulent air are also listed for various conditions.

- Results from some specific calculations are:

Heat Source	Heat flow (Watts)
Interior surface of the dome	-26,000
Conduction from the ground and basement rooms:	
Summer	0 ± 1300
Winter	2600 ± 1300
Thermal inertia of concrete floor and pier	6200
Thermal inertia of the pier alone	5400
Telescope support steel (from M3 study)	11,044

- With assumptions similar to those used by M3, air flow through the dome louvers can carry away 74,000 Watts.
- The e-folding time for the temperature of the steel in the HET structure to reach the ambient air temperature is short, typically 1/2 hour or less.

  The e-folding time scale for the temperature of an uninsulated concrete slab with one exposed wall to reach the temperature of ambient air is

  $$\tau = \frac{\ell}{0.0332} \tag{23}$$

  where ℓ is the thickness of the slab in meters and τ is in hours. Thus, for $\ell = 0.2$ meters, $\tau = 6.0$ hours.

Appendix A: Time Scales for Heat Transfer into Air

A.I. Heat Transfer from One Face of a Flat Slab to Air

Consider a slab with thickness ℓ, cross sectional area A, and mass density ρ. The volume and mass of the slab are

$$\begin{aligned} V &= A\ell & (24)\\ M &= \rho V = \rho A\ell & (25) \end{aligned}$$

The thermal energy of the slab is

$$E = MC_M T_W = \rho A\ell C_M T_W \tag{26}$$

where C_M is the heat capacity per unit mass and T_W is the temperature of the wall. The rate of change of the thermal energy in the slab is, then,

$$\dot{E} = \rho A\ell C_M \frac{dT_W}{dt} \tag{27}$$

The heat carried away from the slab by air is

$$\dot{Q} = -\frac{A}{\mathrm{R}}(T_W - T_A) \tag{28}$$

where T_A is the temperature of the ambient air and the other symbols have their usual meanings. Equating the decrease in the internal energy with the heat flow from the slab to air, we have

$$\begin{aligned} \rho A\ell C_M \frac{dT_W}{dt} &= -\frac{A}{\mathrm{R}}(T_W - T_A) \\ \rho\ell C_M \mathrm{R}\frac{dT_W}{dt} + T_W &= T_A & (29) \end{aligned}$$

The solution to this differential equation is

$$T_W - T_A = \Delta T_0 \, \exp\left(-\frac{t}{\rho\ell C_M \mathrm{R}}\right) \tag{30}$$

where ΔT_0 is an integration constant and is equal to the initial difference between the wall temperature and the air temperature. Equation 30 shows that T_W approaches T_A exponentially with an e-folding time of $\rho\ell C_M\mathrm{R}$.

Example: For concrete exposed to outside air, the values of the constants are

$$\begin{aligned} \rho &= 2800 \text{ kg/m}^3 \\ C_M &= 880 \text{ Joules/kg/}^\circ\text{C} & (31)\\ \mathrm{R} &= 0.044 \text{ mks units} \end{aligned}$$

Inserting these values into equation 30 and converting time from seconds to the more intuitive hours, we have

$$T_W - T_A \;=\; \Delta T_0 \;\exp\left(-\frac{0.0332}{\ell}\,t\right) \tag{32}$$

with t in hours and ℓ in meters. For convenience we set

$$\tau \;=\; \frac{\ell}{0.0332} \tag{33}$$

and then

$$T_W - T_A \;=\; \Delta T_0 \;\exp\left(-\frac{t}{\tau}\right) \tag{34}$$

We recognize τ as the e-folding equilibration time. Equilibration times for some of the concrete components of the HET dome are given in the following table.

Table A1: E-folding Equilibration Times for Concrete in the HET Dome

Component	Thickness (m)	τ (hrs)
Floor inside telescope pier	0.089	2.7
Floor outside telescope pier	0.20	6.0
Telescope Pier	0.5	15

A.II. Heat Transfer from a Cylindrical Beam to Air

Consider a beam with length L, a circular cross section with diameter D, and a mass per unit length σ. The mass and surface area of the beam are:

$$M \;=\; \sigma L \tag{35}$$

$$A \;=\; \pi D L \tag{36}$$

The thermal energy of the beam is

$$E \;=\; M C_M T_W \;=\; \sigma L C_M T_W \tag{37}$$

where C_M is the heat capacity per unit mass and T_W is the temperature of the beam. The rate of change of the thermal energy in the beam is, then,

$$\dot{E} \;=\; \sigma L C_M \frac{dT_W}{dt} \tag{38}$$

Equating the decrease in the internal energy with the heat flow from the beam to air, we have

$$\begin{aligned} \sigma L C_M \frac{dT_W}{dt} &= -\frac{\pi D L}{\mathrm{R}}(T_W - T_A) \\ \frac{\sigma \mathrm{R} C_M}{\pi D}\,\frac{dT_W}{dt} + T_W &= T_A \end{aligned} \tag{39}$$

The solution to this differential equation is

$$T_W - T_A \;=\; \Delta T_0 \;\exp\left(-\frac{\pi D}{\sigma \mathrm{R} C_M}\, t\right) \;=\; \Delta T_0 \;\exp\left(-\frac{t}{\tau}\right) \tag{40}$$

where τ is now

$$\tau \;=\; \frac{\sigma \mathrm{R} C_M}{\pi D} \tag{41}$$

The integration constant ΔT_0 is the initial difference between the wall temperature and the air temperature.

Example: The density and heat capacity of steel are

$$\begin{aligned} \rho &= 7800 \text{ kg/m}^3 \\ C_M &= 460 \text{ Joules/kg/}^\circ\text{C} \end{aligned} \tag{42}$$

The major structural members of the telescope are steel tubes with properties given in Table A2. The time scales for equilibration given in the last column of Table A2 are calculated assuming R = 0.044. They are all short and suggest that the structural steel in the dome follows the ambient air temperature fairly well.

Table A2. Properties of Steel Pipe in the HET Structure

Specification	OD (inches)	ID (inches)	σ (kg/m^2)	τ (hrs)
8” SSP	8.625	7.981	42.52	0.37
8” XSP	8.625	7.625	64.62	0.57
12” SSP	12.750	12.000	73.81	0.43

Appendix B: Major Concerns and Suggested Experiments

- How good is the R-value for air films on surfaces inside the dome? A simple experiment to answer this question: Set a bar of aluminum in the dome where it is well ventilated but not not able to radiate into the night sky. The bar should be round, ∼ 4” in diameter and ∼ 4 ft long. Measure the temperature of the bar and of the ambient air near the bar. Compare the rate of change of the bar’s temperature to the rate of change of the air temperature.

  An alternative experiment: Measure the temperature of the major structural members of the telescope at points half way between the primary mirror and the tracker.

- What are the characteristics of the radiative cooling of the dome? Several experiments suggest themselves:

  Get thermal images of the outside and inside of the dome under various conditions.

  Insulate several adjacent panels of the dome (including the ribs if possible) and take thermal images of the panels from the outside of the dome.

  Put mylar film on the outside of the dome on uninsulated panels of the dome and take thermal images of the panels from the inside of the dome.

- What are the characteristics of air flow outside the dome? Observe smoke trails outside the dome under a wide variety of conditions. This should be done for air near the ground and also for air flowing around the rotating part of the dome.

June 25, 2002

First Recommendations for Improving the Thermal Properties of the HET Environment

E. Barker & E. Robinson

- Increase the insulation on the dome floor by a factor of two to reduce conduction from warm areas underneath the floor.
- Insulate the ceiling and walls of all basement rooms to reduce conduction of heat from the basement to the dome. Even an inch of foam insulation will be more than enough.
- Insulate the pier, including the top of the pier. The pier is the dominant source of heat flow from concrete into the dome during the winter months. The walls of the pier should have as much insulation as the dome floor; do the best we can on the top of the pier.
- We assume that it will be impossible to fully insulate the top of the pier, so the pier needs some active cooling. We recommend installing a cooling collar around the pier near the level of the dome floor. A cooling collar will reduce the effects of thermal inertia in the pier by bringing it closer to the temperature of the ambient air and will choke off heat flow from the ground outside the pier. The choke collar need not have much cooling capacity; 2000 Watts should be more than enough.
- Insulate the dome interior. Cold air pouring down from the dome can be a major source of bad seeing in the dome.
- Reduce radiative cooling of the dome. Especially when the interior of the dome is insulated, the exterior will cool far below the ambient air temperature. Some of the cold air flowing off the outside of the dome will inevitably get into the light beam of the telescope.